FATIGUE ANALYSIS

Two Approaches to Fatigue Life Assessment under Random Vibration

This section was added at the request of our colleagues to address a perfectly reasonable question: why do we need another method for fatigue life assessment, and why do we use it?

Today, the Rainflow Counting Method (or the 'pagoda' method) is effectively the worldwide standard for processing random loading histories. It is used in the aerospace, automotive, energy, and many other industries.

Its popularity is due to its high efficiency. The algorithm makes it possible to reliably identify individual loading cycles from a time history, including cycles with very long periods, and to transform the original random process into a cycle histogram, in which each stress amplitude sa is associated with its number of occurrences n. For most practical applications, this approach is convenient and provides a high level of computational accuracy.

So the question naturally arises: if the Rainflow method works well, why use another method?

I will first give a short answer and then try to explain it in more detail.

The method [1] that we frequently use in our work was developed to assess the fatigue life of lightweight-alloy structures under random loading and was originally developed for aerospace applications.

It does not involve any schematization (i.e., simplification) of the random process.

The actual loading process is considered as a whole rather than as a set of discrete cycles. Its integral characteristics are used to assess fatigue damage. These characteristics are not compared with a fixed S-N curve obtained from laboratory specimens; instead, they are compared with a fatigue curve whose position is adjusted to account for the specific features of the actual random loading process.

This approach is not intended to replace the Rainflow method, nor is it universal for all materials and loading conditions. However, for a range of problems involving random loading of aluminum-alloy structures, it makes it possible to account directly for all the specific features of the loading process that, in conventional processing based on discrete cycles, are taken into account only indirectly.

An additional advantage is the reduction in computational effort achieved by eliminating the need to construct and subsequently process a cyclogram, which is particularly convenient when analyzing stationary random processes.

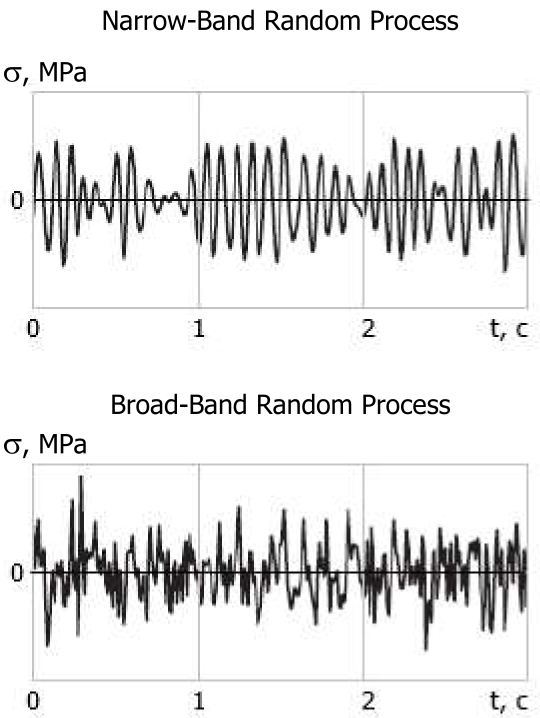


Figure 1. Random Stress Processes

Figure 1 shows two random stress processes with approximately the same number of cycles and approximately the same local maximum and minimum values.

Despite their apparent similarity, the first process (narrowband) is more damaging from a fatigue standpoint due to the characteristics of its internal structure. After applying the Rainflow method to both processes, they produce fatigue cycle distributions (cyclograms) that are similar in shape, so this representation no longer fully reflects the differences between the original processes.

Some information about their statistical structure is inevitably lost, effectively reducing the random process to a simple block loading spectrum. For broad-band processes, there is also the additional problem of correctly accounting for the mean stress in the identified cycles. This, in turn, requires the calculation of equivalent amplitudes using one of the established criteria for accounting for the effect of mean stress, such as Goodman, Gerber, SWT, and others.

To avoid complicating the following discussion, let us consider the simple narrow-band random loading process shown in Figure 2.

Such a process is characteristic of certain structural components of the second and third stages of commercial launch vehicles. Since aerodynamic and acoustic loads are practically absent at this stage of flight, the stress variations are determined primarily by the structural vibration response to the operation of the main engines. For the process under consideration, the mean stresses of individual fatigue cycles are close to zero.

Figure 2. Random Stress Process (Recording Fragment)

This makes it possible to exclude the effect of mean stress from further consideration and focus on the structural characteristics of the random process that directly determine the rate of fatigue damage accumulation.

The average frequency characteristics of the process shown in Figure 2 are presented in Table 1.

Table 1. Random Process Characteristics (see Figure 2)

σ_a^{peak}, MPa	SD, MPa	f_{eff}, Hz	I	N, cycles
150	44	197	0.80	50000

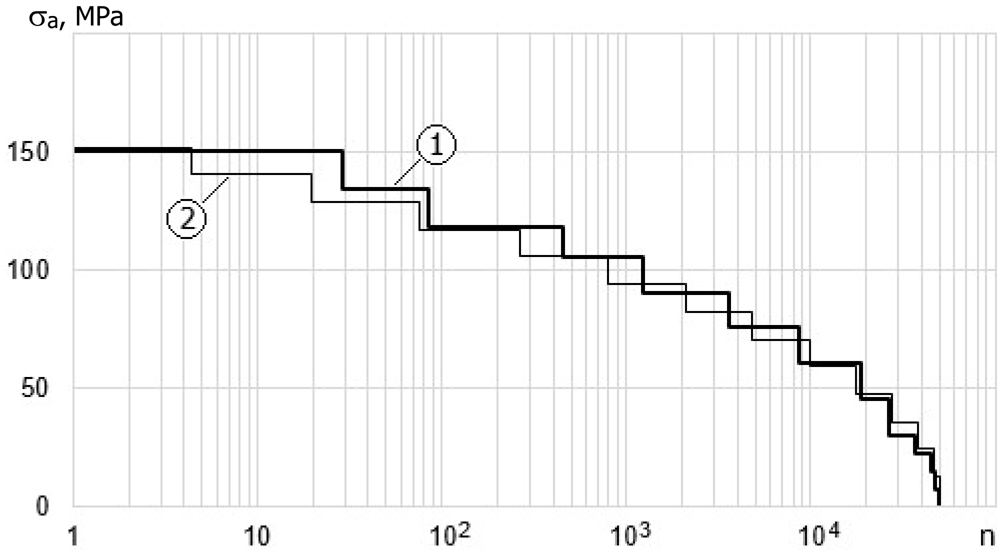
Let us construct two fatigue loading spectra. The first is obtained by processing the time history (see Figure 2) using the standard random-signal processing program 'rainflow.exe'.

The 'rainflow.exe' program divides the entire stress-amplitude range into 13 discrete levels (see Figure 3). Therefore, the second loading spectrum, which was constructed using the mean characteristics of the process given in Table 1, employs the same level of discretization.

A comparison of the results is shown in Figure 4.

Range (units)	Cycle Counts	Ave Amp	Max Amp	Ave Mean	Min Valley	Max Peak
271.5 to 301.7	29.5	141.4	150.9	-3.894	-161.3	140.5
241.4 to 271.5	57.0	129.7	134.3	-5.409	-161.3	140.5
211.2 to 241.4	95.0	114.6	118.4	3.443	-125.7	129.5
181.0 to 211.2	276.5	98.71	105.5	-1.573	-111.0	107.3
150.8 to 181.0	779.0	81.74	90.07	0.5704	-94.89	96.9
120.7 to 150.8	2347.5	66.45	75.42	0.2204	-81.71	84.4
90.5 to 120.7	5140.5	51.6	60.32	-0.183	-67.18	66.35
60.3 to 90.5	10032.0	36.8	45.25	-0.213	-54.42	53.29
45.2 to 60.3	8113.0	26.3	30.16	-0.165	-41.20	38.81
30.1 to 45.2	9643.5	18.91	22.63	-0.086	-38.65	32.99
15.0 to 30.1	8037.0	11.61	15.06	-0.110	-22.46	22.41
7.5 to 15.0	3078.0	5.682	7.538	-0.325	-12.47	13.03
0.0 to 7.5	1796.5	2.091	3.692	0.0706	-10.10	10.36
	n		σ_a	σ_m		

Figure 3. Rainflow Counting of a Random Stress Signal



1 – obtained using the rainflow counting method from the time history
2 – obtained from AFC using the mean characteristics of the process

Figure 4. Fatigue Cycle Distribution (Cyclogram)

The good agreement between the two cyclograms confirms that the average statistical characteristics of the process are sufficient to reproduce the entire fatigue loading spectrum with reasonable accuracy. At this point, we could stop and conclude that identical cyclograms should ultimately result in the same accumulated fatigue damage. Let us confirm this by calculation and carry out the fatigue life calculation to completion for both cases in order to compare the results.

Figure 5 shows the results of fatigue tests on polished laboratory specimens with a diameter of 7.5mm, made of an aluminum alloy. The specimens were subjected to pure bending.

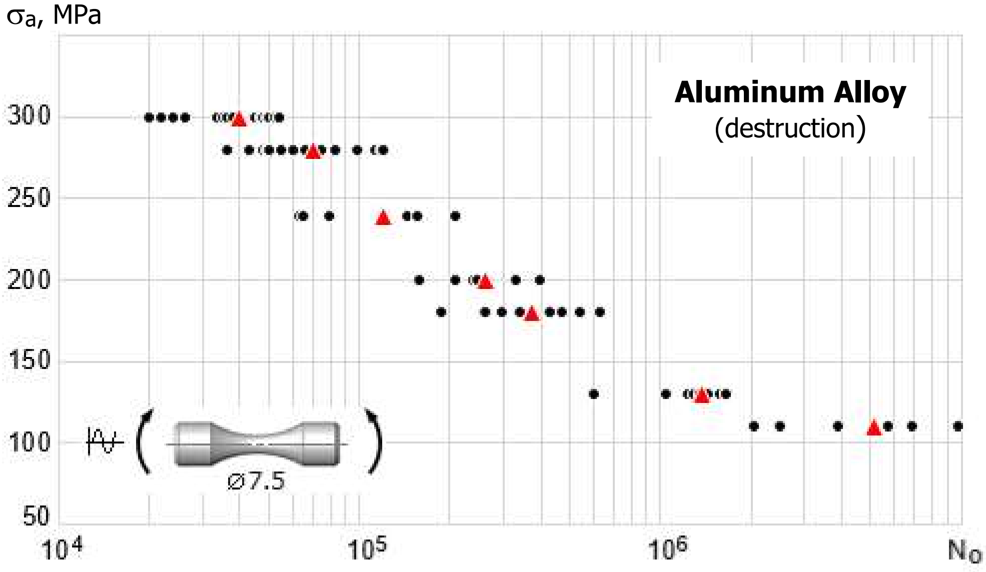


Figure 5. Specimen Fatigue Test Results

Following the recommendations for fatigue analysis using the rainflow method, the experimental data (see Figure 5) are described by the classical S-N curve equation:

$$N = N_k \cdot \left[\frac{\sigma_a}{\sigma_k} \right]^{-k}$$

where:

N_k and σ_k – coordinates of the 'knee point' of the curve;

k and k' – tilt angle indicators before and after the 'knee point'.

$$k' = 2 \cdot k - 2$$

The resulting curve and its characteristics are presented in Figure 6 as a dotted line.

Next step, the same test results are described using an S-N curve equation that includes a fatigue threshold parameter. The amplitudes of the harmonic loading are first converted into the standard deviation (SD) of the process using the following equation:

$$SD = \frac{\sigma_a}{\sqrt{2}};$$

so that the ordinate represents the process SD rather than the stress amplitude σ_a. The resulting curve and its parameters are also shown in Figure 7 as a dotted line.

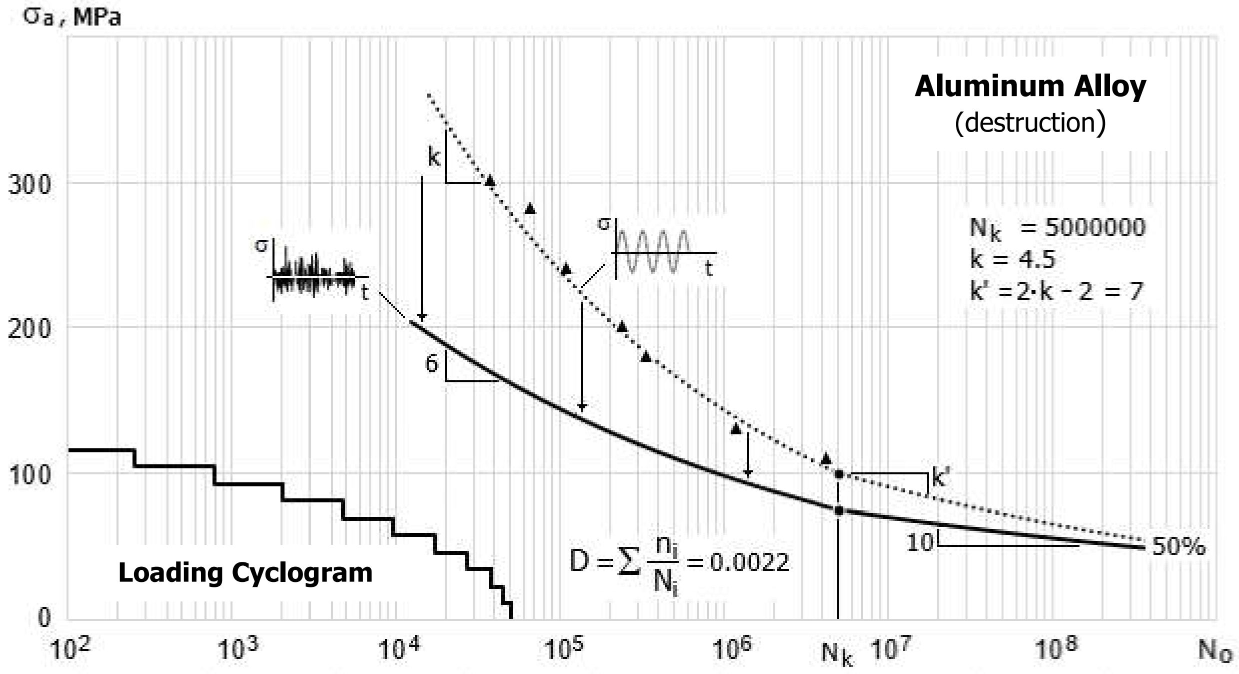


Figure 6. Accumulated Fatigue Damage – Rainflow Counting

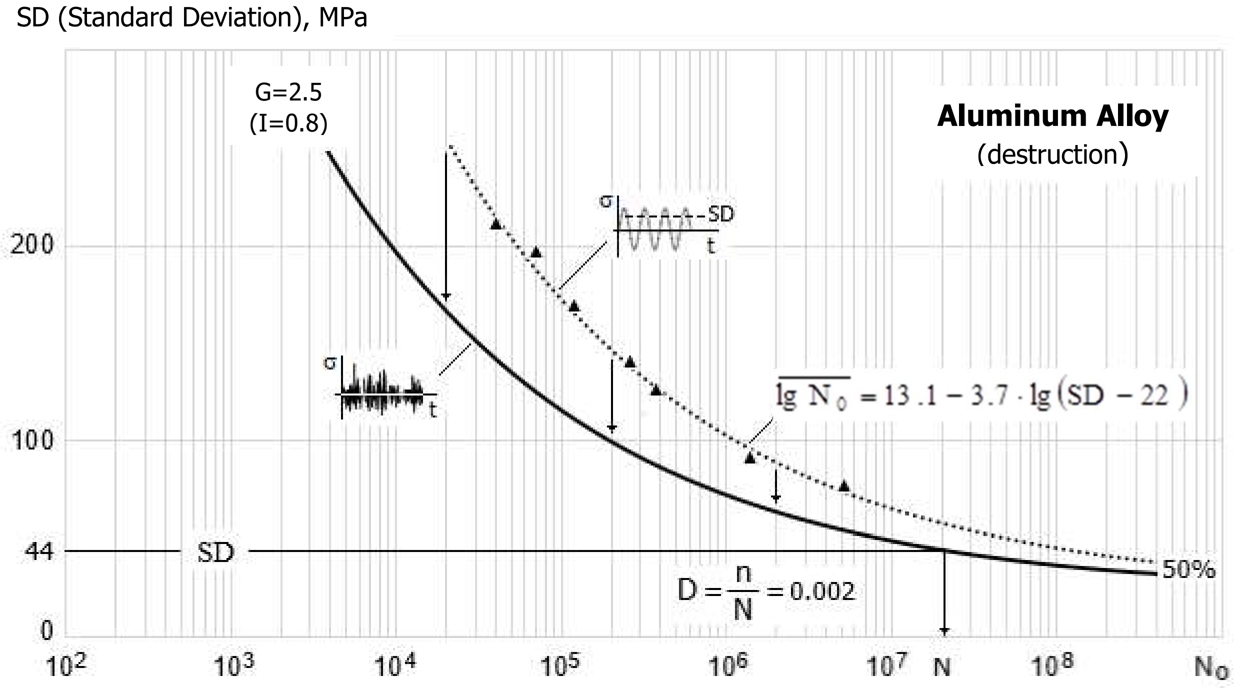


Figure 7. Accumulated Fatigue Damage – Generalized Fatigue Diagram of Material

Let us calculate the accumulated fatigue damage using the two S-N curves corresponding to harmonic loading of the specimens. The resulting D_{harm} values are given in Table 2. The figures also show that, when using the generalized fatigue diagrams, the calculation can be performed in a single step.

Table 2. Damage Calculation

Approach	D_{harm} (50%)	Difference	D_{random} (50%)	Difference
Rainflow Method	0.00041	4%	0.0022	10%
Spectral Method	0.00043		0.0020	

Despite the fundamentally different ways of describing the loading process (SD vs. Amplitude Spectrum), the difference between the results was only 4%.

It was expected because the two approaches are essentially different ways of solving the same problem. However, the values obtained cannot be regarded as a realistic estimate of the accumulated fatigue damage. The process under consideration is random, whereas both S-N curves correspond to harmonic stress variations in the structure. Therefore, the resulting estimate is likely to be significantly underestimated.

The reason is the different rates of fatigue damage accumulation under harmonic and random loading. At the same loading intensity, any narrowband random process generates fatigue damage faster than harmonic loading. Knowing this, the next step is to adjust the material's fatigue limit state to reflect the actual nature of the loading process.

Let's start with the two-parametric method [1].

The location of the fatigue curve is adjusted based on the integral structural characteristics of the random process. The algorithm used by the program is based on the following equation:

$$G = \frac{\sigma_{max}}{СКО} \cdot \frac{RMS_{max}}{RMS_{(+)}} \cdot \frac{n_{max}}{n_{(+)}};$$

and the program can use the following inputs to solve it:

– the time history fragment of the stress process (if the process is stationary);
– the amplitude spectrum of fatigue loading;
– the cyclogram.

An example of the G calculation, along with explanations of all variables in the equation, is provided in Table 4.

Table 4. Structural Parameter G Calculation

Symbol	Value	Unit	Comments
σ_{max}	150	MPa	Maximum Stress in the Time History
СКО	44	MPa	Process Standard Deviation
σ_m	0	MPa	Mean Stress
$n_{(+)}$	50000	cycle	Number of Positive Peaks
n_{max}	32000	cycle	Number of Damaging Peaks (above σ_{-1} = 31MPa)
$RMS_{(+)}$	45	MPa	RMS of Positive Peaks
RMS_{max}	53	MPa	RMS of Damaging Peaks
G	2.5		Structural Parameter

For use in fatigue analysis, the structural parameter G is converted to the dimensionless coefficient ExPC, which quantifies the relative increase in fatigue damage relative to a reference loading process.

The reference is taken to be harmonic loading, for which the value of G is constant and equal to $\sqrt{2}$. The increase in the damaging effect of a random loading process relative to the harmonic one is determined as follows:

$$\mathrm{ExPC} = \frac{G_0 - \sqrt{2}}{G_0 - G};$$

where G_0 is a material constant. For most aluminum alloys, its value lies within the range of 3.7...4.3. In practical applications, G_0 = 4.0 is generally adopted. However, the selection of its exact value depends on the material and on accumulated experience with the proposed methodology.

To account for the effect of random loading, the position of the material S-N curve is adjusted according to the calculated ExPC value using the following equation:

$$\overline{\lg N_0} = 13.1 - 3.7 \cdot \lg\left[\mathrm{ExPC} \cdot (\mathrm{SD} - 22)\right];$$

The resulting curve is shown as a solid line in Figure 7, and the calculated values of the accumulated damage, D_{random}, are given in Table 2.

In the classical approach based on the Rainflow method, there is no universal algorithm for making such an adjustment to the S-N curve. In practice, the effect of random loading is usually taken into account indirectly by changing the slope parameter k of the fatigue curve.

The recommended values are based on accumulated experience in analyzing particular classes of structures rather than on the statistical characteristics of the specific loading process.

For comparison, we will use k = 6, as this value is most commonly recommended for the analysis of structures made of aluminum alloys. The corresponding curve is shown as a solid line in Figure 6.

The difference between the results obtained using the two approaches was about 10%, which might suggest good agreement between the methods. However, the close agreement between the two results does not mean that both approaches will provide a similar level of accuracy for other random loading processes.

In the present example, k = 6 proved to be a fortunate choice and provided good agreement between the results. However, this parameter was selected based on general recommendations for aluminum alloys and is in no way related to the statistical structure of the loading process being analyzed. If the load spectrum changes or a different structure is considered, the degree of agreement may change significantly.

This is the fundamental difference between the two approaches. In the classical method, the effect of random loading is accounted for through an empirical selection of the fatigue-curve parameters. In the two-parameter approach, the adjustment is based on the characteristics of the loading process itself, making it possible to directly account for its damage-producing properties.

There is no need to examine the resulting error further. The purpose of this comparison is to show that, when analyzing random loading, it is not sufficient to use the original S-N curve obtained from laboratory specimen tests. Regardless of the calculation method, the material fatigue limit state must be adjusted to reflect the nature of the actual loading process. Only then can calculated fatigue-life estimates be expected to correlate satisfactorily with test results and experimental data.

Conclusion:

If we accept that both methods can produce the same amplitude spectrum, and that accurate calculation of accumulated fatigue damage, taking into account the characteristics of the random loading process, requires the material fatigue curve to be recalibrated (rebuilt) using theoretically or experimentally determined coefficients, then the need to convert a random process into a load spectrum (or cyclogram) becomes questionable.

These merely change the way the loading information is represented, while inevitably losing some of that information. It leads back to the point made at the beginning: schematization (simplification) of actual random loading processes may simply be an unnecessary mathematical operation.

This approach is not new and has been used for several decades to assess the fatigue life of structures operating under random loading.

For structures made of aluminum and magnesium alloys, the use of statistical characteristics of the stress process in combination with generalized fatigue diagrams makes it possible to obtain sufficiently accurate – and, when necessary, conservative – estimates of accumulated fatigue damage over the required service period.

List of Sources

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