**FATIGUE ANALYSIS**

# Accounting for Structural Differences in Fatigue Analysis

For accurate evaluation of the fatigue life and safety margins of structural elements, the material's fatigue properties (usually derived from the S-N curve of laboratory specimens) should be corrected. This correction involves accounting for the differences between the actual structural component and the laboratory specimen, including the effects of stress concentration, component size, type of loading, surface hardening, surface finish, etc.

The following question is considered using a wheel rim as a case study: Is the fatigue-life prediction based on the specimen S-N curve conservative?

The fatigue test results for the specimens are presented in Figure 1 as dots. The S-N curve represents the 50% probability of failure (i.e., the mathematical expectation of a specimen fatigue failure).

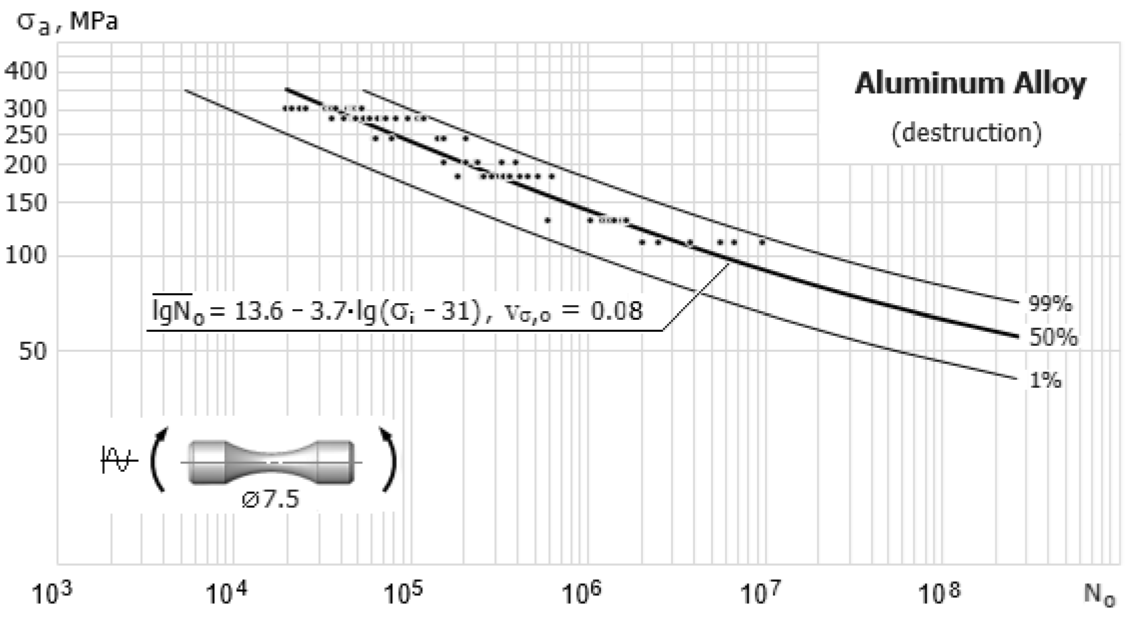



Figure 1. S-N Curve of the Aluminum Alloy

Specimens are cut directly from the wheel rim, near the hub, and below the fatigue critical section (see Figures 2, 7, and 9). This means that the wheel rim and the specimens were produced using the same technology and have similar mechanical, but different fatigue properties.

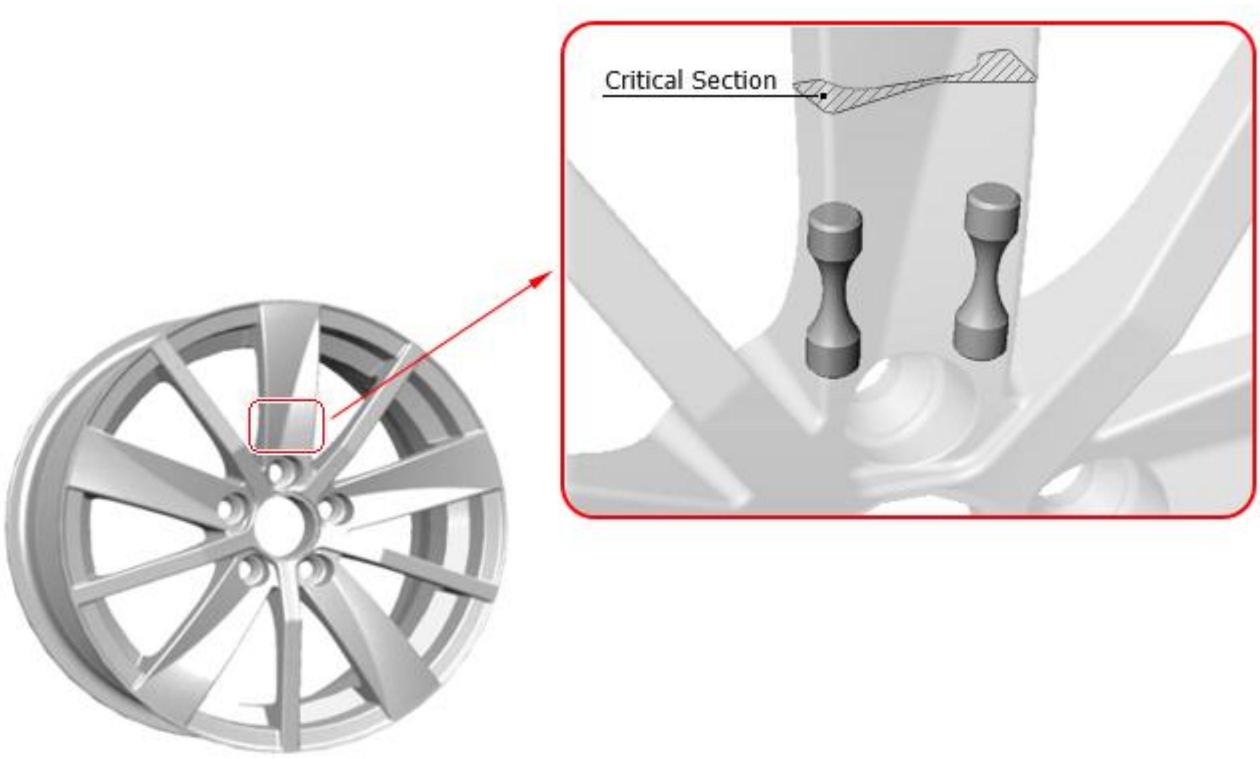



Figure 2. Specimens Cutting Area

The difference in fatigue properties between a real structural element and a specimen is due to several factors:

- component size;
- type of loading;
- stress concentration;
- surface finishing or coating.

If we assume that the surface finishing is identical, then the last factor from the list can be eliminated. In this case, the method suggested by S. Menshikov [1] can be utilized to consider the effect of the first three factors on the fatigue limit. This method is based on the calculation of the coefficient $K_\sigma$, which accounts for the combined influence of stress concentration, component size, and type of loading on the S-N curve location.

$$K_\sigma = 1 + t_o \cdot \nu_{\sigma,o} \left[ \left( \frac{I_o}{I_d} \right)^{\frac{1}{m}} - 1 \right]; \qquad (1)$$

where:

$I_o$ – I-integral of the specimens that were used for the material S-N curve creation;

$\nu_{\sigma,o}$ – coefficient of variation of the specimens fatigue limits;

$I_d$ – I-integral of the analyzed element;

$m$ and $t_o$ – parameters of the probability distribution for random variables, the following values can be used for calculations: $m$ = 4 and $t_o$ = 3.56.

To account for structural differences, the coefficient $K_\sigma$ is introduced into the specimen S-N curve equation:

$$\overline{\lg N_{0i}} = A - B \cdot \lg\left[\frac{1}{K_\sigma}\sigma_{ai} - \sigma_0\right];$$

where: $A$ and $B$ – coefficients of the S-N curve;

$\sigma_0$ – mean value of the 'endurance limit'.

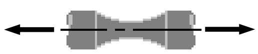
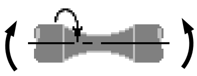
The coefficient of variation of the fatigue limit has also been corrected and reflects the dispersion of the fatigue properties of the analyzed element, not the specimens:

$$\nu_{\sigma,d} = \frac{t_o \cdot \nu_{\sigma,o} + K_\sigma - 1}{t_o \cdot K_\sigma}. \tag{2}$$

To determine these coefficients, it is necessary to calculate the values of two integrals: $I_o$ and $I_d$ (for the specimen and the wheel spoke fragment, respectively).

The I-integrals corresponding to standard fatigue specimens have analytical solutions, some of which are presented in Table 1 below.

Table 1. Formulas for Determining I-integrals of Simple Specimens

| Specimens and Type of Loading | | I - integral |
|---|---|---|
| Tension-compression |  | $I_o = b \cdot h \cdot (t_o \cdot \nu_\sigma)^4$ |
| |  | $I_o = \frac{\pi \cdot d^2}{4}(t_o \cdot \nu_\sigma)^4$ |
| Pure Bending |  | $I_o = \frac{b \cdot h}{5}(t_o \cdot \nu_\sigma)^5$ |
| Bending & Rotation |  | $I_o = \frac{\pi \cdot d^2}{60}(6 - t_o \cdot \nu_\sigma)(t_o \cdot \nu_\sigma)^4$ |

For calculating the integrals of more complicated structural elements, a numerical solution method is used.

The cross-section of a wheel rim selected to demonstrate this approach is shown in Figure 7.

First, to perform a detailed analysis, it is necessary to obtain the gradient of the stress distribution along the cross-section passing through the critical node as accurately as possible. There are two methods for doing this: creating a more complicated model in CAE or using the SA program. The latter approach is simpler because the cross-section can be modeled independently of the GFEM.

The 3D CAD model is sectioned by a plane passing through the selected node, orthogonally to the plane of action of the maximum principal stresses. The cross-section is then meshed using finite elements, with a focus on ensuring that all elements are of the same size. Modern software algorithms make this process easy (see Figure 3).

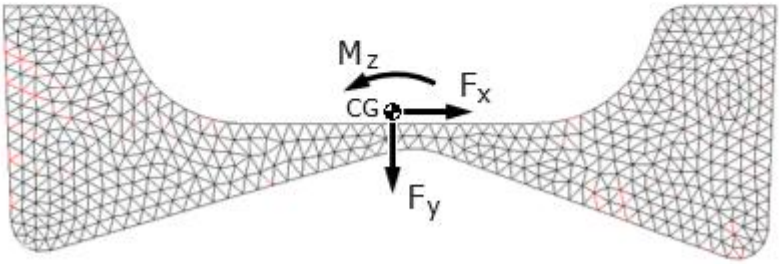
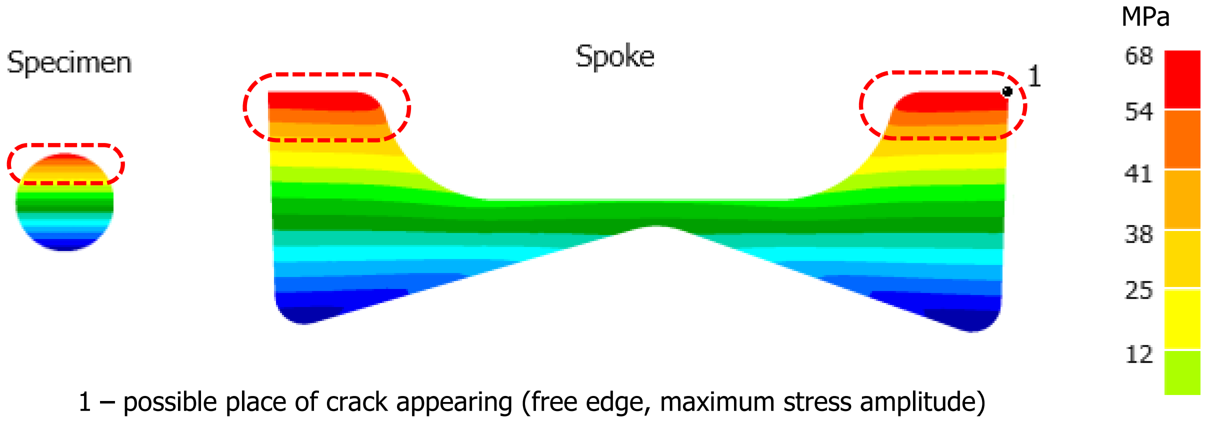



Figure 3. Critical Section meshing and loading

The forces and moments applied to the cross-section are obtained from the preliminary GFEM analysis (MSC.PATRAN option: Grid Point Forces Balance, see Figure 7).

The load case producing the maximum stress amplitude in the critical section should be selected for further analysis.

The calculated stress contours (isolines) for the two critical sections are shown in Figure 4.




1 – possible place of crack appearing (free edge, maximum stress amplitude)

Figure 4. Stress Distribution in the Fatigue Critical Sections

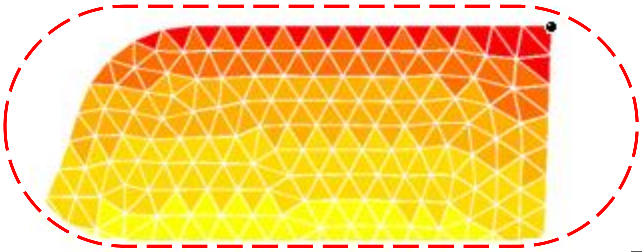


Figure 5. Stress in Elements

Only elements for which the following condition is satisfied: $\sigma_i \geq \sigma_0$ are of interest (the areas are highlighted by red frames in Figure 4).

The normal stress within each element may be assumed to be constant (i.e. $\sigma_i = \text{const}$), and the area $A_i$ may also be treated as constant (see Figure 5).

The most straightforward way to calculate the $I_d$ value from the integral equation of the analyzed section (see Figure 5) is through a numerical technique. Accept:

$$k_i = \frac{\sigma_i}{\sigma_{max}}$$

where: $\sigma_{max}$ – maximum stress acting in the section;

$\sigma_i$ – stress acting in the element.

Replacing the integration over the entire cross-sectional area with summation over discrete finite-element areas $A_i$ yields:

$$I_d = \sum_{i=1}^{n} \left(k_i - 1 + t_o \cdot \nu_{\sigma,o}\right)^4 A_i \qquad (3)$$

where $n$ – number of elements meeting the requirement: $k_i \geq 1 - t_0 \cdot \nu_{\sigma,0}$

Equation (3) provides a simple and efficient means of evaluating the I-integral for structural elements of arbitrary geometry. Preparing the initial data for these calculations is not a difficult task, although accuracy is essential.

After obtaining the normal stresses in the selected elements using the SA program, the terms of Equation (3) are evaluated and summed (see Table 2).

Table 2. Results of Critical Section Analysis

| № | Element | $\sigma_i$ , МПа | $k_i$ | $A_i$, мм$^2$ | $(k_i - 1 + t_o \cdot \nu_{\sigma,o})^4 \cdot A_i$ |
|---|---|---|---|---|---|
| 1 | 352402 | **67.97** | 1.000 | 0.112 | 4.33871E-04 |
| 2 | 352515 | 66.27 | 0.975 | 0.112 | 2.84744E-04 |
| 3 | 352514 | 66.22 | 0.974 | 0.112 | 2.80867E-04 |
| 4 | 352513 | 65.43 | 0.963 | 0.112 | 2.26715E-04 |
| 5 | 352512 | 65.25 | 0.960 | 0.112 | 2.15630E-04 |
| 6 | 352511 | 64.91 | 0.955 | 0.112 | 1.95846E-04 |
| 7 | 352510 | 64.76 | 0.953 | 0.112 | 1.87715E-04 |
| 8 | 352509 | 64.52 | 0.949 | 0.112 | 1.74617E-04 |
| 9 | 352508 | 64.45 | 0.948 | 0.112 | 1.70991E-04 |
| 10 | 352507 | 64.42 | 0.948 | 0.112 | 1.69842E-04 |
| … | | | | | |
| 257 | 355201 | 51.05 | 0.753 | 0.444 | 1.74224E-14 |
| | | | | **$I_d$** | **0.0107** |

When the $I_d$ value has been obtained, the coefficients $K_\sigma$ and $\nu_{\sigma,d}$ can be calculated as follows (2) and (3):

$$K_\sigma = 1 + 3.564 \cdot 0.07\left[\left(\frac{0.0022}{0.0107}\right)^{\frac{1}{4}} - 1\right] = 0.91,$$

$$\nu_{\sigma,d} = \frac{3.564 \cdot 0.080 + 0.91 - 1}{3.564 \cdot 0.91} = 0.060$$

For a better understanding, all data is collected in Table 3.

Table 3. S-N Curve Parameters

| № | Section | $K_\sigma$ | $\nu_\sigma$ | Base S-N Curve | S-N Curve |
|---|---|---|---|---|---|
| 1 | Specimen | 1.0 | 0.08 | $\overline{\lg N_0} = 13.60 - 3.7\lg\left(\frac{1}{K_\sigma}\sigma_i - 31\right)$ | $\overline{\lg N_0} = 13.60 - 3.7\lg(\sigma_i - 31)$ |
| 2 | Spoke | 0.91 | 0.06 | | $\overline{\lg N_0} = 13.45 - 3.7\lg(\sigma_i - 28)$ |

The figure below shows the change in the fatigue curve location relative to the base one.

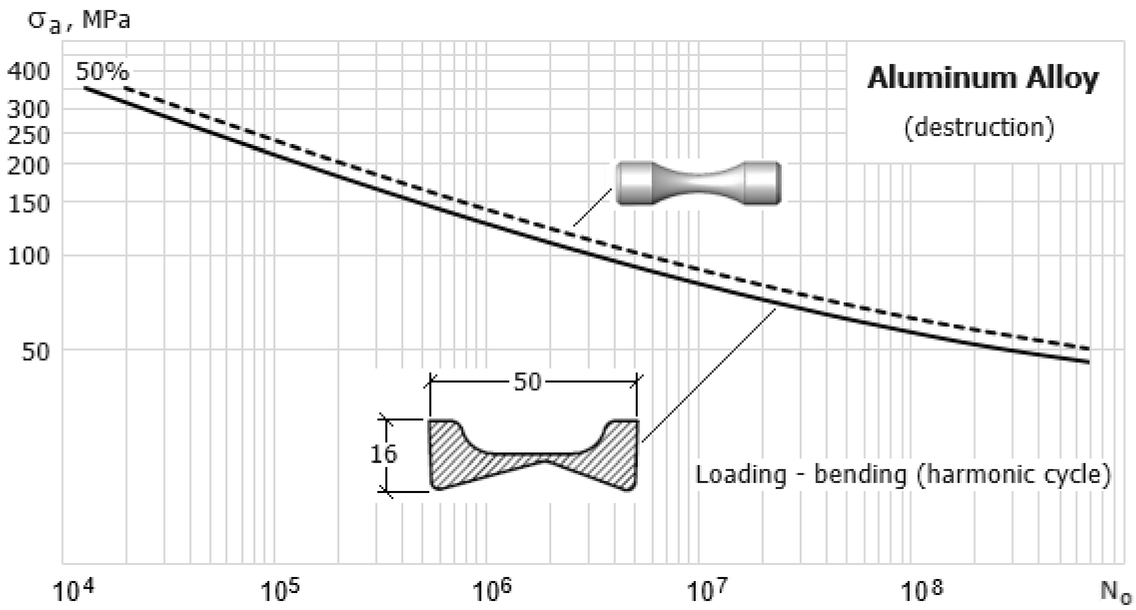



Figure 6. The S-N Curves

**Conclusion:**

The corrected S-N curve is shifted to the left (i.e., toward shorter fatigue lives). Therefore, using the specimen S-N curve for fatigue analysis of the wheel spokes would result in a non-conservative (overestimated) fatigue life.

The above-described theory is mathematically simple and can be readily implemented in software.

The corrected fatigue properties are not used directly as individual S-N curves. Instead, they are stored as additional parameters of the generalized fatigue diagram and subsequently used for fatigue analysis of actual structural components (see Table 4).

Table 4. Parameters of the Generalized Fatigue Diagram for an Aluminum Alloy

| # | Sample | A | B | $\sigma_o$ | $K_\sigma$ | $v_\sigma$ | C | $s_o$ | $G_o$ | G | $C_{surf}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Type I, round 7.5mm, bending | 13.6 | 3.7 | 31 | 1.00 | 0.08 | 17.45 | 0.052 | 3.8 | 1.41 | 1.00 |
| 2 | Spoke#1, bending | 13.6 | 3.7 | 31 | 0.91 | 0.06 | 17.45 | 0.001 | 3.8 | 1.41 | 1.00 |

## Limitations of the Analysis

The present analysis does not account for the influence of the surface finish factor ($C_{surf}$ = 1.0). Depending on the manufacturing process, surface treatment may either improve or reduce fatigue performance. Evaluation of this factor is outside the scope of the present study.

## References

1. Меньшиков С.Я. Вероятностный метод расчёта характеристик сопротивления усталости элементов конструкций: Автореф. дис. канд. техн. наук – Челябинск, 1989. – 16с.

   //Menshikov, S.Y. Probabilistic Method for Evaluating Fatigue Resistance Characteristics of Structural Components. PhD Thesis Abstract, Chelyabinsk, 1989, 16 p.

APPENDIX

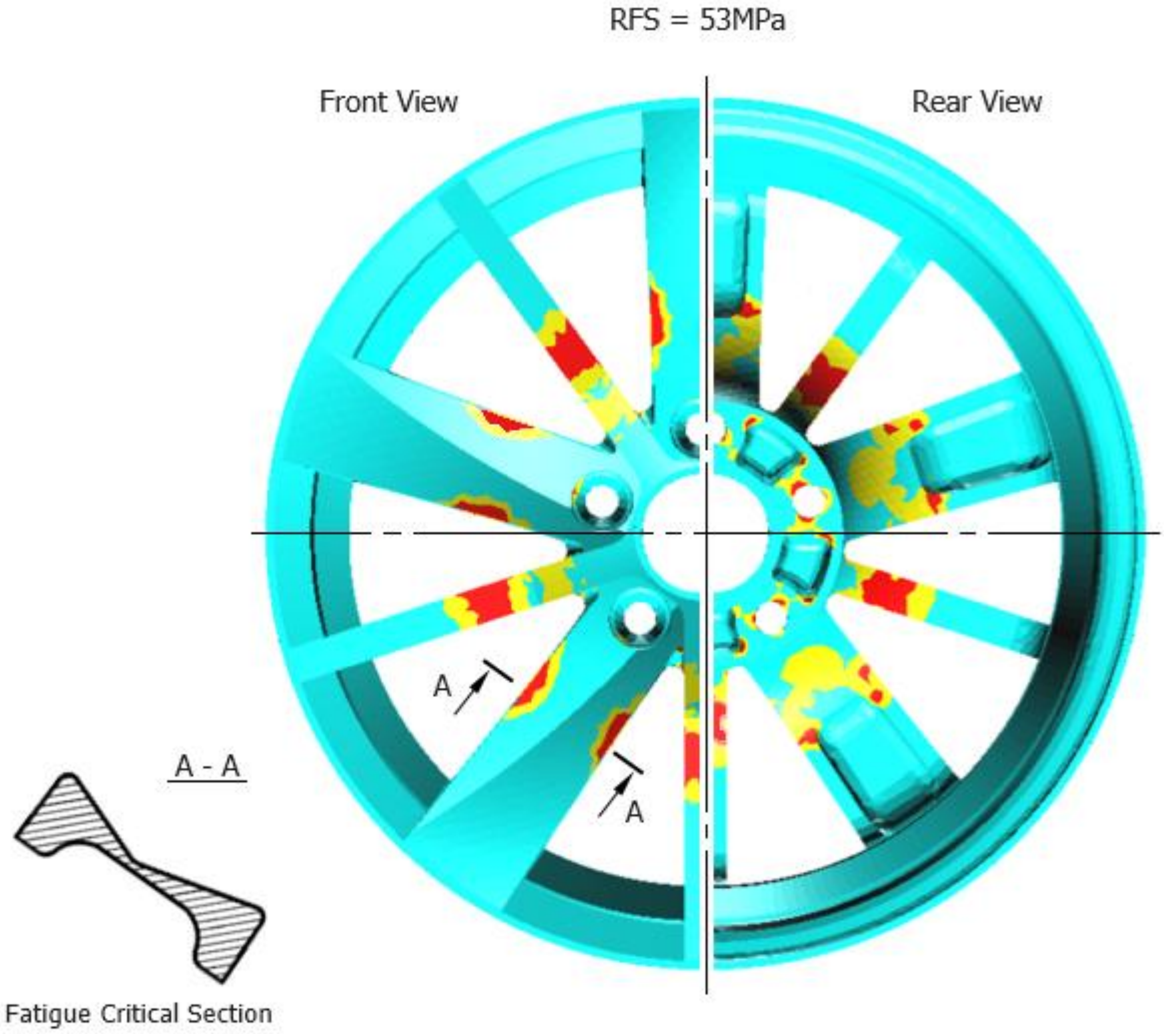



Figure 7. Stress Amplitude Contours above RFS (Required Fatigue Strength)

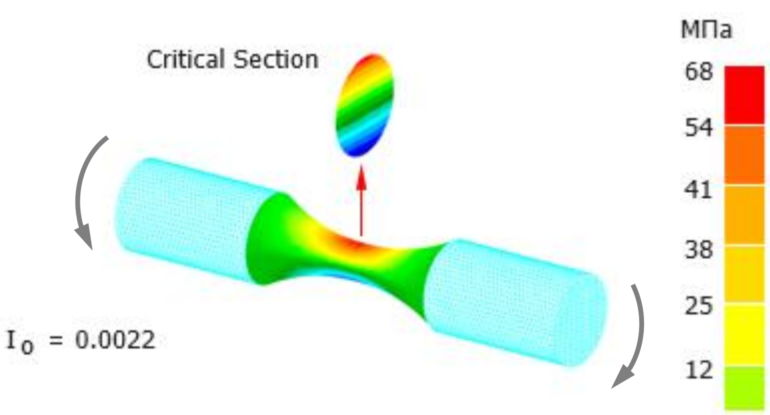



Figure 8. Specimen FEM Analysis


Figure 9. Cutting Samples from Wheel Rim