FATIGUE ANALYSIS

The Design Difference Accounting

For accurate evaluation of the fatigue life and safety margins of structural elements, the fatigue limit value of the material (usually, the S-N curve of specimens is used for this) should be corrected. This correction involves considering the difference between the real structural element and specimen (i.e., accounting for the various factors such as stress concentrator, size, type of loading, surface hardening, surface finishing, …).

Let's try to answer the following question using a wheel rim as an example - what is the accuracy of estimating its fatigue life using the specimen's S-N curve (conservative or not)?

The fatigue test results for the specimens are presented in Figure 1 as dots. The S-N curve represents the 50% probability of failure (i.e., the mathematical expectation of a specimen fatigue failure).

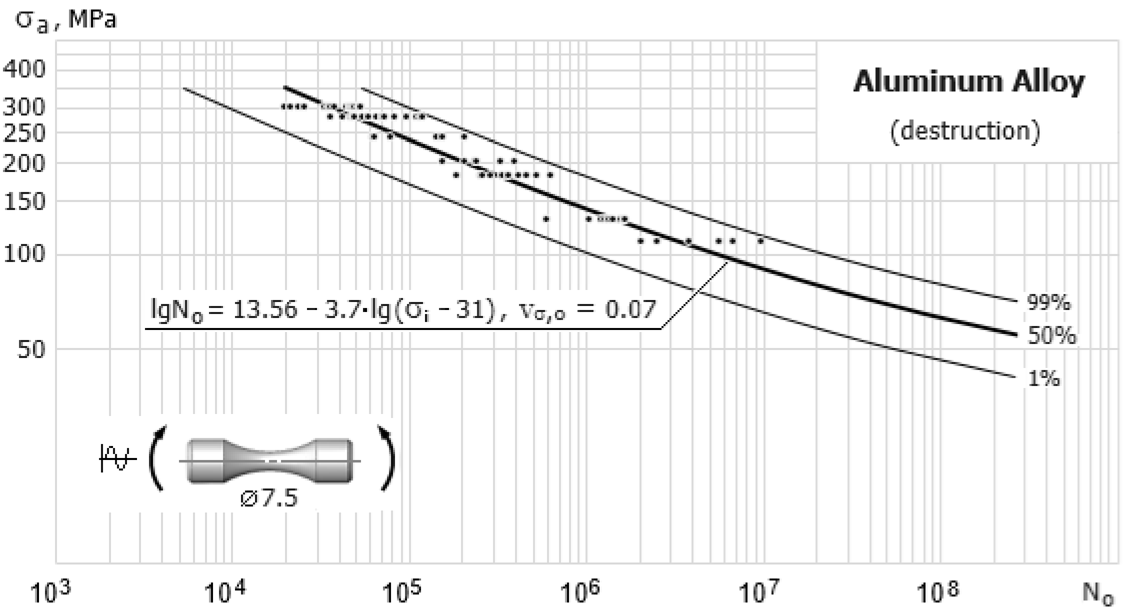


Figure 1. The S-N Curve of Aluminum Alloy

Specimens are cut directly from the wheel rim, near the hub, and below the fatigue critical section (see Figures 2, 7, and 9). This means that the wheel rim and the specimens were produced using the same technology and have similar mechanical, but different fatigue properties.

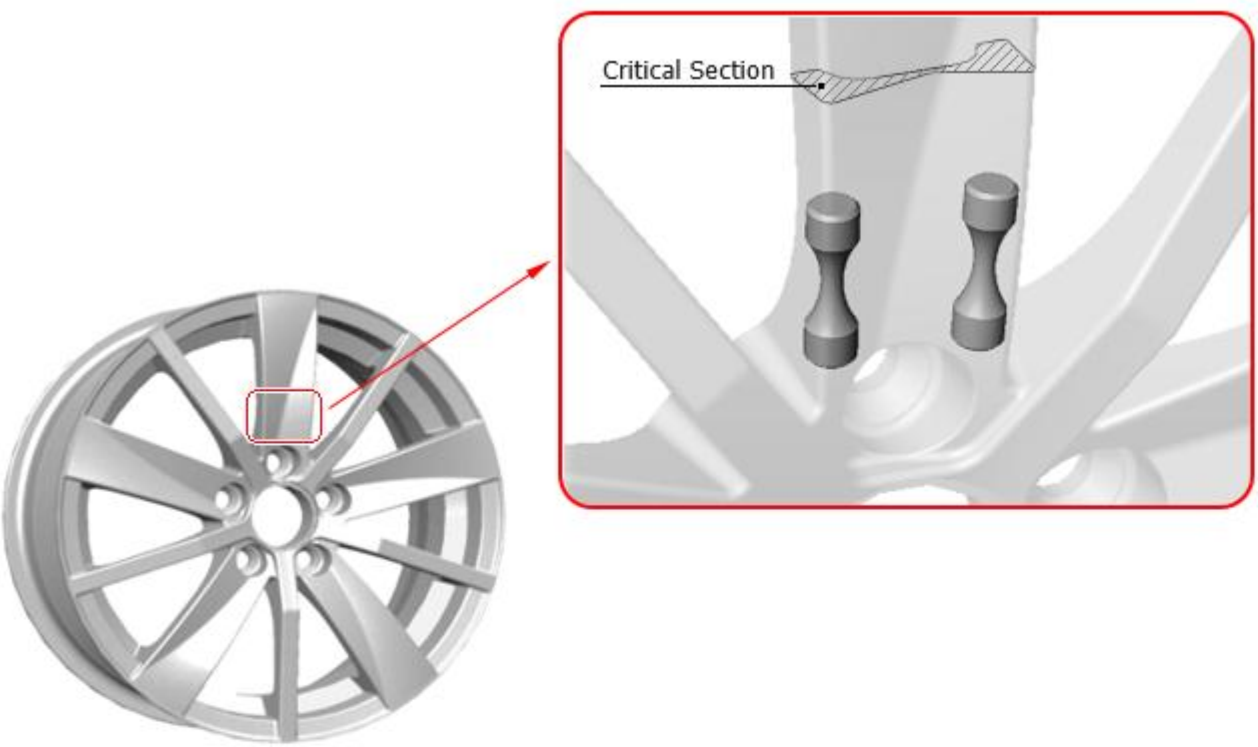


Figure 2. Specimens Cutting Area

The difference in fatigue properties between a real structural element and a specimen is due to several factors:

- size;
- type of loading;
- stress concentration;
- surface finishing or coating.

If we assume that the surface finishing is identical, then the last factor from the list can be eliminated. In this case, the method suggested by S. Menshikov [1] can be utilized to consider the effect of the first three factors on the fatigue limit. This method is based on calculation of the K_σ coefficient, which is complex and reflects the combined influence of the stress concentration, size and type of loading on the S-N curve location.

$$K_\sigma = 1 + t_o \cdot \nu_{\sigma,o} \left[\left(\frac{I_o}{I_d} \right)^{\frac{1}{m}} - 1 \right]; \qquad (1)$$

where:

I_o – I-integral of the specimens that were used for the material S-N curve creation;

$\nu_{\sigma,o}$ – coefficient of variation of the specimens fatigue limits;

I_d – I-integral of the analyzed element;

m and t_o – parameters of the probability distribution for random variables, the following values can be used for calculations: m = 4 and t_o = 3.56.

To account for the differences, the K_σ parameter is introduced into the specimen S-N curve equation as an additional coefficient:

$$\overline{\lg N_{0i}} = A_h - B \cdot \lg\left[\frac{1}{K_\sigma} \cdot \sigma_i - \sigma_{-1}\right];$$

where: A and B – coefficients of the S-N curve;

σ_{-1} – endurance limit.

The coefficient of variation of the fatigue limit has also been corrected and reflects the dispersion of the fatigue properties of the analyzed element, not the specimens:

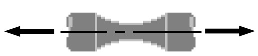
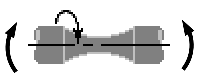
$$\nu_{\sigma,d} = \frac{t_o \cdot \nu_{\sigma,o} + K_\sigma - 1}{t_o \cdot K_\sigma}. \tag{2}$$

To determine these coefficients, it is necessary to calculate the values of two integrals: I_o and I_d (for the specimen and the wheel spoke fragment, respectively).

The I-integrals for specimens, which are often used in fatigue testing, have analytical solutions, some of which are presented in Table 1 below.

Table 1

Formulas for Determining I-integrals of Simple Specimens

Specimens and Type of Loading		I - integral
Tension-compression		$I_o = b \cdot h \cdot (t_o \cdot \nu_\sigma)^4$
		$I_o = \frac{\pi \cdot d^2}{4}(t_o \cdot \nu_\sigma)^4$
Pure Bending		$I_o = \frac{b \cdot h}{5}(t_o \cdot \nu_\sigma)^5$
Bending & Rotation		$I_o = \frac{\pi \cdot d^2}{60}(6 - t_o \cdot \nu_\sigma)(t_o \cdot \nu_\sigma)^4$

For calculating the integrals of more complicated structural elements, a numerical solution method is used.

The cross-section of a wheel rim selected to demonstrate this approach is shown in Figure 7.

First, to perform a clarifying analysis, it is necessary to obtain the gradient of the stress distribution along the cross-section passing through the critical node as accurately as possible. There are two methods for doing this: creating a more complicated model in CAE or using the SA program. The latter is a bit easier, due to the cross-section can be created separately from the GFEM.

The 3D CAD model is cut by a plane passing through the selected node, orthogonally to the plane of action of the maximum principal stresses. Then, the cross-section is meshed by finite elements, with a focus on ensuring that all elements are of the same size. Modern software algorithms make this process easy (see figure below).

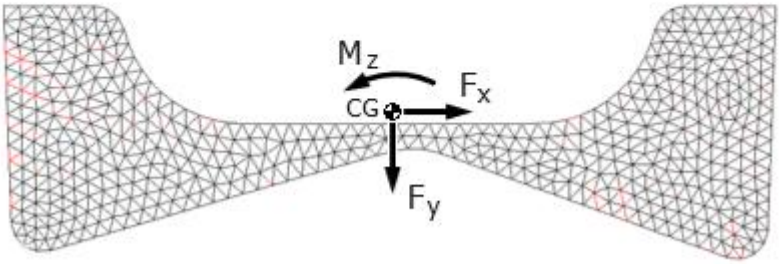
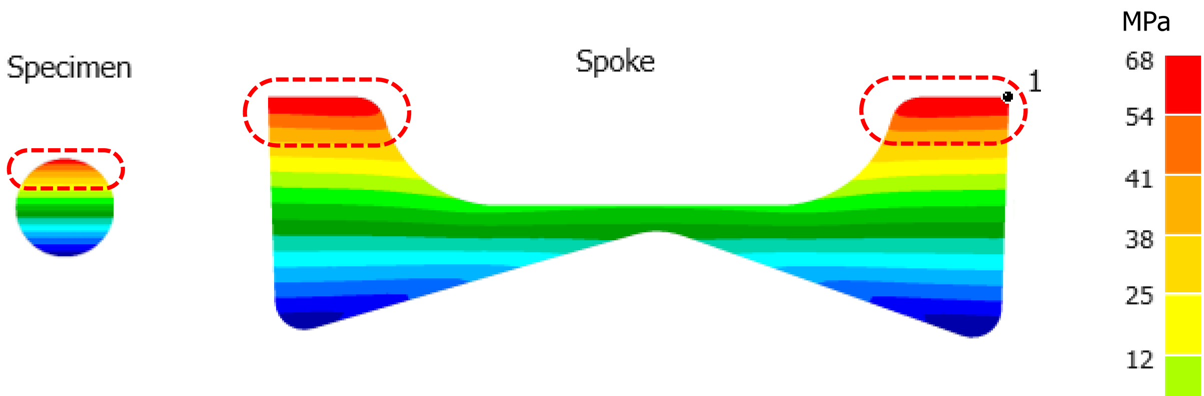


Figure 3. Critical Section meshing and loading

Forces and moments in the loading cross-section are derived from the results of the preliminary GFEM analysis (MSC.PATRAN option: Grid Point Forces Balance, see Figure 7).

As an analysis case, it is better to select loadcase that produces the maximum stress amplitude at the critical cross-section.

The calculated stress contours (isolines) for the two critical sections are shown in the figure below.



1 – possible place of crack appearing (free edge, maximum stress amplitude)

Figure 4. Stress Distribution in the Fatigue Critical Sections

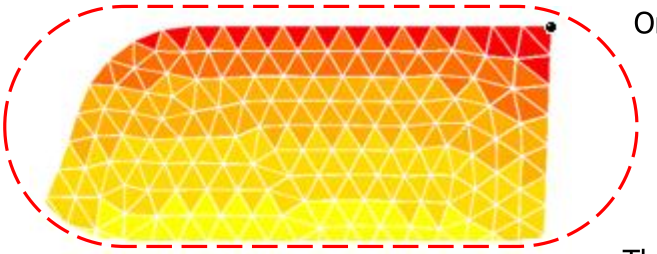


Figure 5. Stress in Elements

Only elements in which the acting stresses comply with requirement: $\sigma_i \geq \sigma_{-1}$ are of interest (the areas are highlighted by red frames in Figure 4).

The normal stresses within each element can be considered approximately constant (i.e. $\sigma_i = \text{const}$), as well as the area of each element A_i (see. Figure 5).

The most straightforward way to calculate the I_d value from the integral equation of the analyzed section (see Figure 5) is through a numerical technique. Accept:

$$k_i = \frac{\sigma_i}{\sigma_{max}}$$

where: σ_{max} – maximum stress acting in the section;

σ_i – stress acting in the element.

If change integration over the entire cross-sectional area with sum of the discrete areas A_i, obtain:

$$I_d = \sum_{i=1}^{n} \left(k_i - 1 + t_o \cdot \nu_{\sigma,o}\right)^4 \cdot A_i \tag{3}$$

where n – number of elements meeting the requirement: $k_i \geq 1 - t_0 \cdot \nu_{\sigma,0}$

Equation (3) allows calculating the I-integral value for any structural element shape rapidly and straightforwardly. Preparing the initial data for these calculations is not a difficult task, although accuracy is essential.

After obtaining the normal stresses in the selected elements using the SA program, the coefficients for equation (3) are computed and then added together (see Table 2).

Table 2

Results of Critical Section Analysis

№	Element	σ_i , МПа	k_i	A_i, мм^2	$(k_i - 1 + t_o \cdot \nu_{\sigma,o})^4 \cdot A_i$
1	352402	**67.97**	1.000	0.112	4.33871E-04
2	352515	66.27	0.975	0.112	2.84744E-04
3	352514	66.22	0.974	0.112	2.80867E-04
4	352513	65.43	0.963	0.112	2.26715E-04
5	352512	65.25	0.960	0.112	2.15630E-04
6	352511	64.91	0.955	0.112	1.95846E-04
7	352510	64.76	0.953	0.112	1.87715E-04
8	352509	64.52	0.949	0.112	1.74617E-04
9	352508	64.45	0.948	0.112	1.70991E-04
10	352507	64.42	0.948	0.112	1.69842E-04
…					
257	355201	51.05	0.753	0.444	1.74224E-14
				I_d	**0.0107**

When the I_d value has been obtained, the coefficients K_σ and $\nu_{\sigma,d}$ can be calculate as follows (2) and (3):

$$K_\sigma = 1 + 3.564 \cdot 0.07 \cdot \left[\left(\frac{0.0022}{0.0107} \right)^{\frac{1}{4}} - 1 \right] = 0.92 ,$$

$$\nu_{\sigma,d} = \frac{3.564 \cdot 0.070 + 0.91 - 1}{3.564 \cdot 0.91} = 0.051$$

For a better understanding, all data is collected in Table 3.

Table 3

The S-N Curves Parameters

№	Section	K_σ	ν_σ	Base S-N Curve	S-N Curve
1	Specimen	1.0	0.07	$\overline{\lg N_o} = 13.56 - 3.7 \lg\left(\frac{1}{K_\sigma} \cdot \sigma_i - 31 \right)$	$\overline{\lg N_o} = 13.56 - 3.7 \lg(\sigma_i - 31)$
2	Spoke	0.92	0.051		$\overline{\lg N_o} = 13.39 - 3.7 \lg(\sigma_i - 28)$

The figure below shows the change in the fatigue curve location relative to the base one.

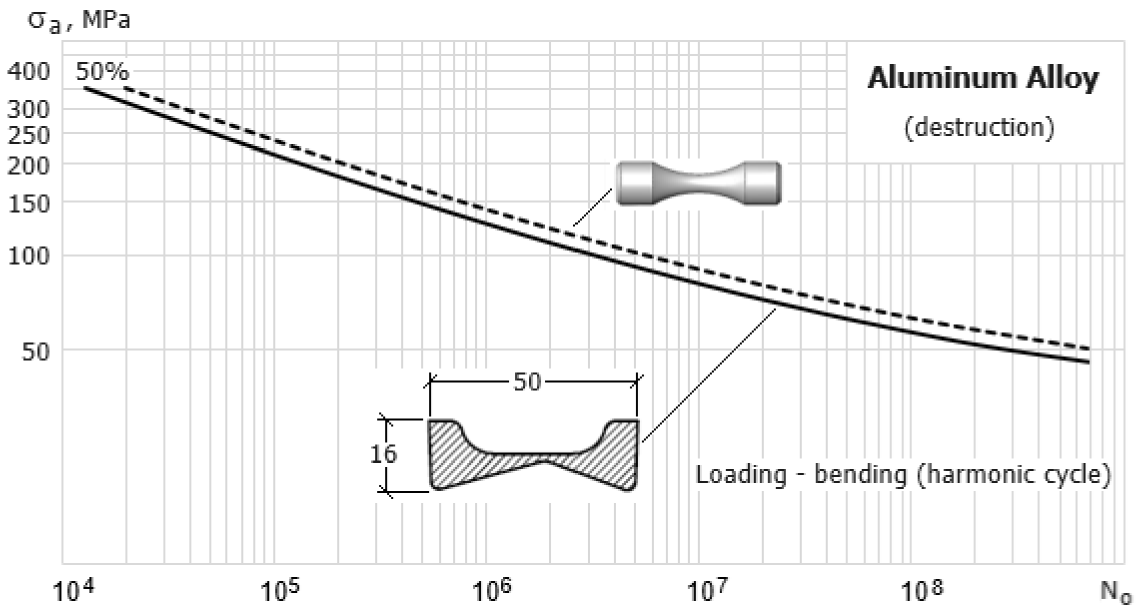


Figure 6. The S-N Curves

Conclusion:

The curve has shifted to the left. That means, if the specimen's S-N curve is used for calculation of the wheel spoke fatigue life, the result will be overestimated.

The above-described theory is mathematically simple and can be easily algorithmized for macro programming.

p.s.

The analysis is not yet completed, as an assumption was made: the surface finishing factor (C_{surf}) is not included. It can improve or significantly worsen fatigue properties, but this is not the topic of this article

List of Sources

1. Меньшиков С.Я. Вероятностный метод расчёта характеристик сопротивления усталости элементов конструкций: Автореф. дис. канд. техн. наук – Челябинск, 1989. – 16с.

APPENDIX

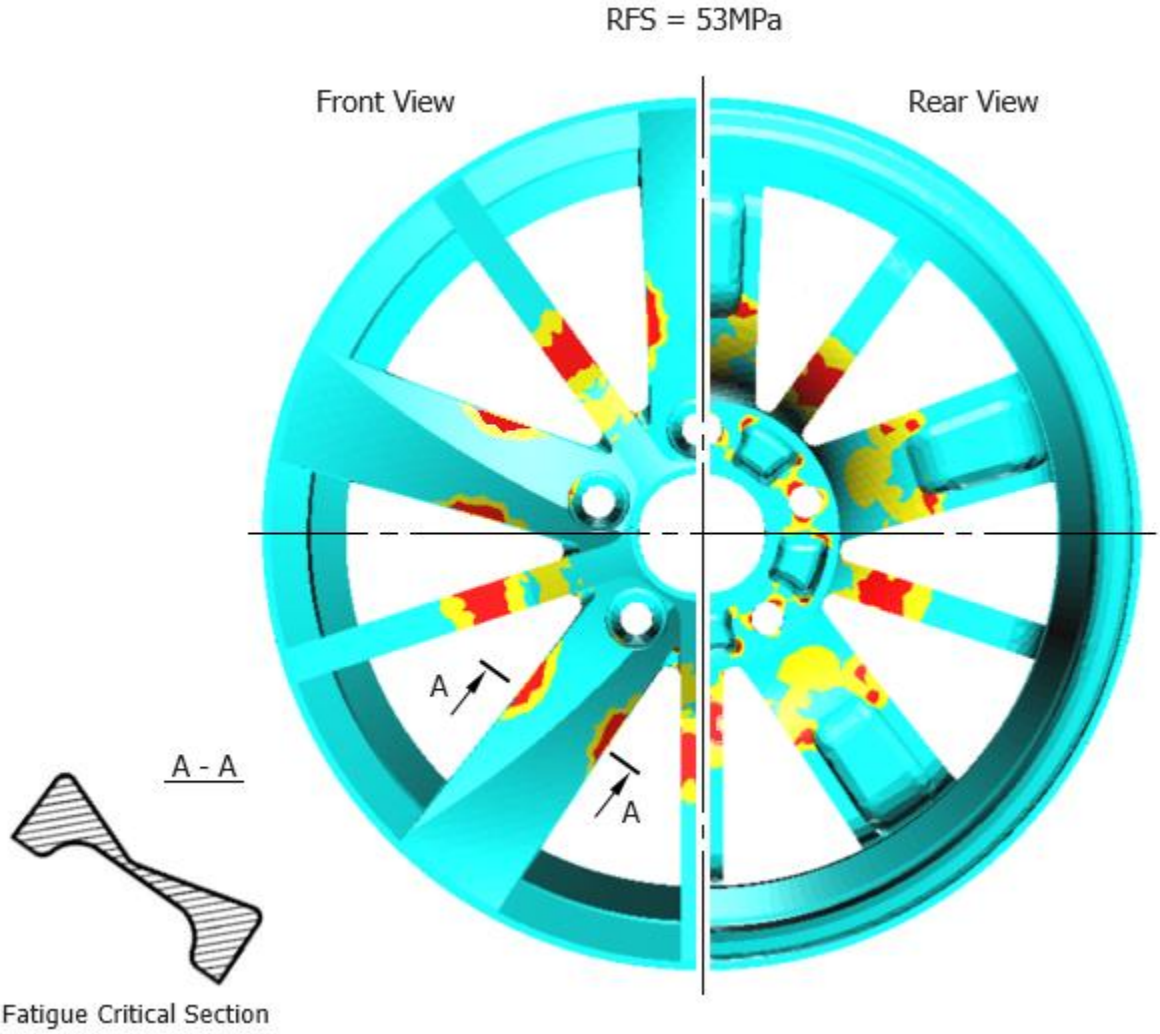


Figure 7. Stress Amplitude Contours above RFS (Required Fatigue Strength)

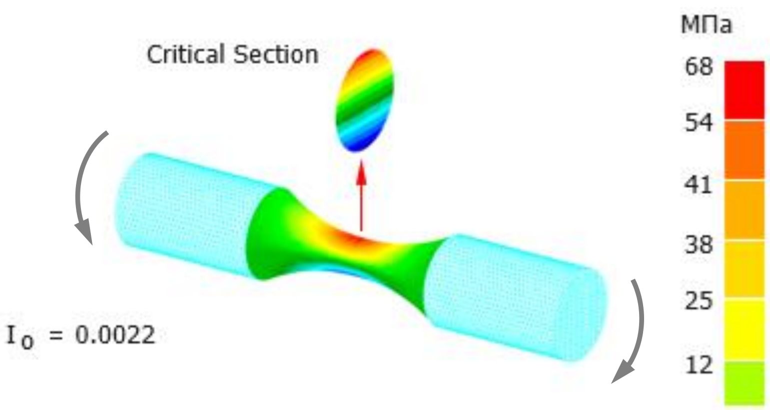


Figure 8. Specimen FEM Analysis

Figure 9. Cutting Samples from Wheel Rim