FATIGUE ANALYSIS

EVALUATION OF THE FATIGUE LIFE OF A STRUCTURE UNDER RANDOM VIBRATION

When translating a number of our articles into English, we found it necessary to clarify several basic terms that are commonly understood without further explanation in Russian-language technical literature.

The fatigue limit state of a material is defined by its fatigue curve, the classical equation of which mathematically relates two parameters: the cyclic stress amplitude and fatigue life (σ_a vs. $\lg N$).

However, when evaluating the fatigue life of structures operating under random vibration, this method of representation becomes insufficient.

The approach frequently used in our work is not conventional*. It is based on four numerical characteristics of a random process that describe both the intensity of material loading and the structure of the process in terms of its influence on fatigue life. These characteristics are:

- σ_m – the mean value of the stress process, characterizing the static stress offset;
- SD – the standard deviation of stress, characterizing the loading intensity;
- N_0 – the number of zero-level intersections of the centered process with a specified derivative sign, which determines the fatigue life;
- G – a criterion describing the probabilistic structure of the loading process.

Of these four parameters, only G requires additional explanation. It is an integral characteristic of the damaging part of the loading process. However, since only basic concepts are being introduced at this stage, a detailed discussion of this parameter will be postponed.

* This approach is not widely used in engineering practice due to its relatively narrow field of application. It has been developed primarily for the analysis of lightweight-alloy structures subjected to random loading, particularly in the aerospace industry. Before the advent of reusable systems, such problems were encountered relatively infrequently, which limited the demand for this calculation method. Nevertheless, its effectiveness has been confirmed not only by experimental testing but also by many years of successful operational experience with structures for which fatigue failure would have unacceptable consequences. Thus, the reliability of fatigue-life predictions has been demonstrated not only by test data but also by the successful operation of systems designed using this methodology.

To describe the fatigue limit state of a material subjected to random loading, it is necessary to establish relationships involving not two traditional parameters, but four. The input parameters are the mean stress value σ_m, the standard deviation SD, and the structural criterion G, while the output parameter characterizing fatigue damage is the number of cycles N_0 or lgN_0.

To visualize such relationships, a three-dimensional space σ_m – SD – lgN_0 was proposed, in which the position of a fatigue surface is determined by the structure of the loading process. Each value of the structural criterion G corresponds to its own fatigue surface (see Figure 1).

In practice, however, the procedure is much simpler. There is no need to construct fatigue surfaces. The entire process has been automated, and existing software packages, such as 'RESURS', automatically determine the fatigue limit state.

Nevertheless, to understand the principles underlying such algorithms, it is useful first to consider a more convenient representation of material fatigue properties for engineering calculations – 'the Generalized Fatigue Diagram of a Material'.

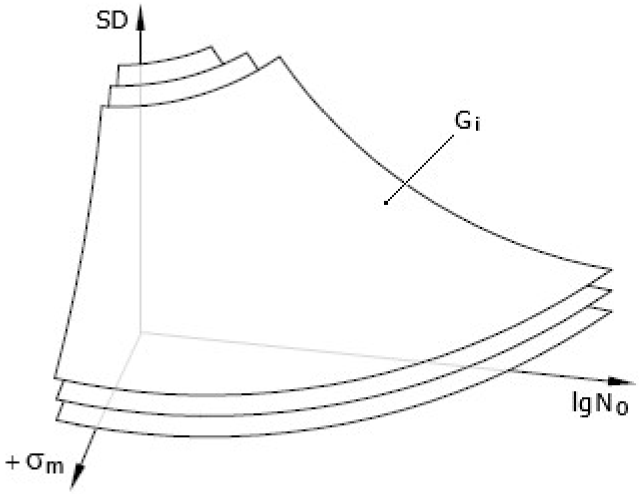
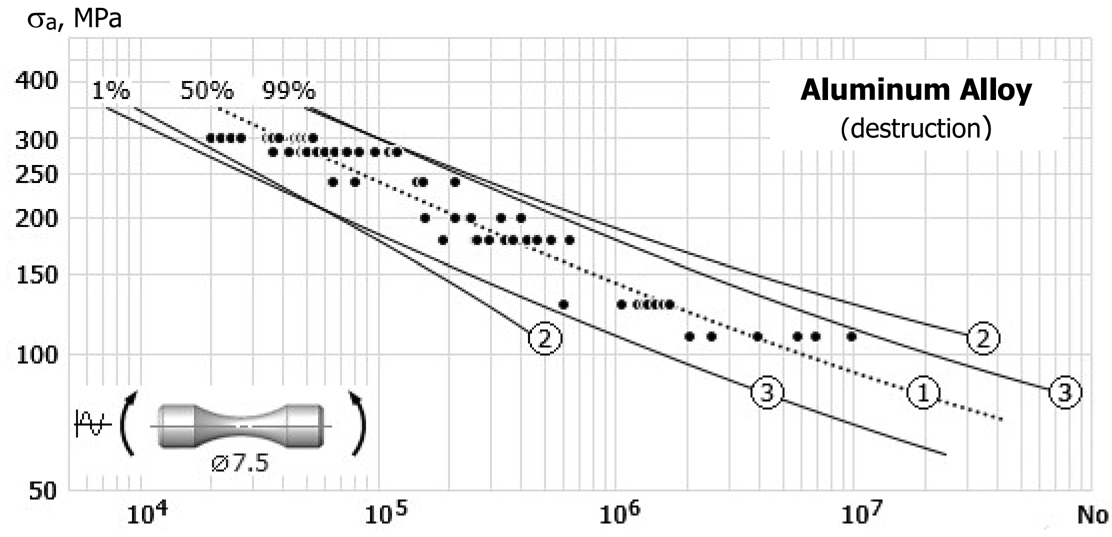


Figure 1. Material Fatigue Surfaces

For simplicity, let us assume σ_m = 0. In this case, the representation becomes the classical two-dimensional one again; however, the statistical description of fatigue test results is fundamentally different.

Generalized Fatigue Diagram of a Material

The generalized fatigue diagram is an approach to the statistical analysis of fatigue test results for specimens made of lightweight alloys, particularly aluminum alloys, according to the traditional Soviet fatigue analysis methodology (see Figure 2).



1 – S-N curve equation (1);
2 – S-N curve equation (2);
3 – S-N curve equation (4).

Figure 2. Generalized Fatigue Diagram of Material

One of the characteristic features of aluminum alloys, in contrast to most structural steels, is the absence of a clearly defined endurance limit (σ_{-1}) corresponding to infinite fatigue life. Fatigue damage continues to accumulate even at relatively low stress levels. To account for this behavior, fatigue test results are commonly described using a bi-parametric Wöhler curve or an S-N curve equation that includes a fatigue threshold parameter:

$$\overline{\lg N}_{0i} = A - B \cdot \lg[\sigma_{ai} - \sigma_0], \tag{1}$$

where:

A and B – parameters of the fatigue curve under harmonic loading;

σ_0 – mean value of the endurance limit**.

The points in Figure 1 represent the results of laboratory tests on polished round specimens. Statistical analysis of the experimental data using the least-squares method yields the following equation:

$$\overline{\lg N}_{0i} = 13.6 - 3.7 \cdot \lg[\sigma_{ai} - 31],$$

which corresponds to a 50% probability of failure (see curve 1 in Figure 1). However, this form of the equation is not used in practical engineering calculations. The reason is obvious: structures are not designed with a 50% probability of failure

* The symbol σ_{-1} is also sometimes used. However, for aluminum alloys, σ_o or σ_{-1} should be regarded primarily as a parameter of the S-N curve equation that defines the material's sensitivity to cyclic loading and represents a conditional asymptote. Formally, stress amplitudes below this threshold should not contribute to fatigue damage, which relates this parameter to the classical definition of σ_{-1}. Unlike fatigue strength at a specified fatigue life, σ_o is not associated with any particular number of cycles. Instead, it is obtained by fitting the S-N curve to experimental data and defines its asymptotic behavior.

The required reliability level must be substantially higher. To achieve this, the statistical scatter of fatigue test results must be taken into account.

The statistical scatter of fatigue test results can be described in two complementary ways.

The first approach is based on analyzing the statistical distribution of specimen fatigue life at a given loading amplitude. In this case, the equation used to describe the test results assumes a normal distribution of the logarithm of fatigue life:

$$\lg N_{0i} = A - B \cdot \lg[\sigma_{ai} - \sigma_0] \pm tq \cdot s_{\lg Ni}, \quad (2)$$

where:

- tq – Student's t-distribution quantile, determined by the required probability and the number of available experimental data points;
- s_{lgNi} – sample standard deviation of lgN_i at a given stress amplitude σ_{ai}.

The value of S_{lgNi} is not constant; it depends on the applied stress level and is inversely related to it. In other words, the lower the stress level, the greater the scatter. Therefore, this parameter is described mathematically using the following equation:

$$S_{\lg N_i} = s_o + \frac{C}{\sigma_{ai} - \sigma_0}, \quad (3)$$

where s_o and C are two material constants obtained from the statistical analysis of the test results. The equation for the present case is shown in Figure 3.

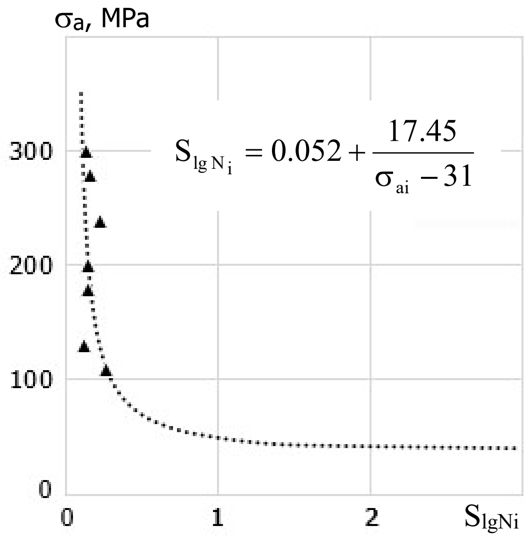


Figure 3. Approximation of the logarithm of the fatigue life distribution

An important feature of the hyperbolic representation of the scatter (see Figure 3) is the existence of a transition region near σ_o, where the calculated scatter begins to increase rapidly and eventually reaches physically unrealistic values. This feature will later help simplify the calculation, as will become clear in the following sections.

For lightweight alloys, equation (2) alone is insufficient to adequately describe the scatter of fatigue test results at low stress amplitudes. Therefore, a second, complementary approach is introduced, based on the statistical scatter of fatigue strength at a given fatigue life.

Equation (4) is used for this purpose. It describes the same fatigue test results from a different statistical perspective, assuming a normal distribution of limited-endurance fatigue strength:

$$\lg N_{0i} = A - B \cdot \lg\left[\frac{1}{1 \pm tq \cdot v_\sigma}\sigma_{ai} - \sigma_0\right], \tag{4}$$

where v_σ – coefficient of variation of limited-endurance fatigue strength (constant).

$$v_{\sigma i} = \frac{S_{\sigma i}}{\bar{\sigma}_{ai}} = \text{const},$$

where:

$S_{\sigma i}$ – sample standard deviation of fatigue strength at a given fatigue life $\lg N_i$;

$\bar{\sigma}_{ai}$ – mean value of fatigue strength at the same fatigue life $\lg N_i$.

For each test level, the value of v_σ is determined separately, after which its mean value is calculated. Statistical analysis of the test results (see Figure 2) showed that this coefficient varies within a range:

$$0.064 < v_\sigma < 0.119;$$

with a mean value of 0.088; for engineering applications, a value of 0.08 was adopted.

Thus, to adequately describe the probability-dependent position of the S-N curve, two parameters are used simultaneously – one variable and one constant:

$S_{\lg Ni}$ – characterizes the scatter of fatigue life at a given stress amplitude (see eq. (2));

v_σ – characterizes the relative scatter of fatigue strength at a given fatigue life (see eq. (4)).

The relationships given by equations (2) and (4) provide a good description of the experimental test data, but each is applicable only within its respective range on the fatigue curve (see Figure 2, curves 2 and 3).

If the number of cycles to failure calculated using equation (2) is denoted as $\lg N_{0i}^{(2)}$, and that calculated using equation (4) as $\lg N_{0i}^{(4)}$, then, for fatigue life calculations, the actual number of cycles to failure under a given loading condition is taken as:

$$\lg N_{0i} = \begin{cases} \max\left\langle \lg N_{0i}^{(4)}, \lg N_{0i}^{(2)} \right\rangle & \text{if } P_s < 0.5 \\ \min\left\langle \lg N_{0i}^{(4)}, \lg N_{0i}^{(2)} \right\rangle & \text{if } P_s > 0.5 \end{cases}$$

where P_s is the probability of failure.

Generalized fatigue diagrams are not normally presented graphically, as shown in Figure 2. Therefore, when a material's generalized fatigue diagram is mentioned, it refers to a table containing a set of parameters for at least three equations: (2), (3), and (4), which describe:

– the position of the S-N curve (eq. (1));
– the scatter of fatigue life;
– the scatter of fatigue strength; and
– the rule for selecting the limiting state for a specified probability of failure (or survival).

An example is given in Table 1.

Table 1. Base Parameters of the Generalized Fatigue Diagram for an Aluminum Alloy*

Sample	A	B	σ_o, MPa	v_σ	C	s_o
Type I, round 7.5mm, bending	13.6	3.7	31	0.080	17.45	0.052

* The original test results should preferably be provided in a separate file as factual confirmation of the validity of the derived coefficients and, more importantly, of the volume of experimental data processed (ten tested specimens are insufficient for constructing a diagram).

An example of the practical application of the generalized fatigue diagram is given in Table 2.

Table 2. Allowable Number of Loading Cycles

#	σ_a, MPa	P_s, %	tq	lgN	Comments
1	300	0.1	3.38	4.21	equation (2)
2	250	1	2.46	4.62	equation (2)
3	200	5	1.69	5.08	equation (4)
4	50	10	1.31	8.43	equation (4)

Table 1 presents the minimum set of material properties required for the analysis. When necessary, it can be supplemented with additional parameters that make it possible to determine the fatigue life limit of the material not only under harmonic loading, but also under random, polyharmonic loading with closely spaced frequencies, and mixed loading, with a specified probability of survival (see Table 3).

Table 3. Parameters of the Generalized Fatigue Diagram of Specimens from an Aluminum Alloy**

#	Sample	A	B	σ_o	K_σ	v_σ	C	s_o	G_o	G	C_{surf}
1	Type I, round 7.5mm, bending	13.6	3.7	31	1.00	0.08	17.45	0.052	3.8	1.41	1.00

** Parameters not described here are used in the random-vibration fatigue algorithm and are explained in the next sections.